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Sheaves of Abelian Groups and the Quotients A^{**}/A

G. SCHLITT*

University College of the Fraser Valley, 33844 King Road, Abbotsford, British Columbia V2S 4N2, Canada

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We extend Mrówka's theorem for groups of continuous functions to groups of global sections of sheaves. As an application we show that for any Abelian group A of non-ω-measurable cardinality there is an Abelian group D so that $D^{**}/D \cong A$. This allows us to answer a question of Eklof and Mekler regarding the groups D^{***}/D^*; it follows that any dual group is of the form D^{***}/D^* for some group D. © 1993 Academic Press, Inc.

1. INTRODUCTION

An important problem in the study of torsion-free Abelian groups concerns the structure of the group $\mathrm{Hom}(A, \mathbf{Z})$ (the *dual* of A, henceforth denoted A^*), and its relation to the structure of the group A. Several long-standing problems of Reid [11] have recently been solved, and the methods of their solution have allowed investigation into deeper aspects of the theory. In this article we extend these methods, and use them to show how the structure of A^{**} relative to A may be arbitrarily complex.

Recall that an (Abelian) group A is said to be reflexive if the canonical map $\sigma: A \to A^{**}$ is an isomorphism, and torsionless if this map is injective. One of Reid's original questions was whether every dual group is reflexive. A negative answer was first shown consistent in [5], and an example constructed in ZFC in [2], where Eda and Ohta used a theorem of Mrówka (Theorem 3.2) to find conditions on a space X equivalent to $C(X, Z)^*$ being non-reflexive, and they found a space X satisfying these conditions. In fact, for this space X, $C(X, \mathbf{Z})^{**} = C(X, \mathbf{Z}) \oplus \mathbf{Z}$, and hence $C(X, \mathbf{Z})^{***} = C(X, \mathbf{Z})^* \oplus \mathbf{Z}$ (where we identify a torsionless group with its image under the σ map). Note that $C(X, \mathbf{Z})^{***}/C(X, \mathbf{Z})^* \cong \mathbf{Z}$ so that the quotient is a dual group. In fact, this is true in general as the following theorem shows. (See [4, p. 423] for a proof.)

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PROPOSITION 1.1. *Given a group* A, *let* $A_n = A^*_{n-1}$, *and* $A_0 = A$. *Then*

(i) A_n *is a direct summand of* A_{n+2} *for* $n \geqslant 1$, *and*

(ii) A_{n+3}/A_{n+1} *is a dual group; in fact the dual of* A_{n+2}/A_n, *for* $n \geqslant 0$.

The results above allow one to refine Reid's question and ask, what dual groups may appear as a quotient A^{**}/A for some dual group A? In [4] it is shown how an answer to this question may be used towards elucidating the structure of dual groups. There it is proved that any *double* dual can appear as such a quotient (assuming some set-theoretic hypotheses) and it is conjectured that any dual group so appears. Here we prove this conjecture, (Corollary 4.5) via a lifting of the theorem of Mrówka mentioned above to the class of groups of global sections (Theorem 3.3). Our main result is stronger than the original conjecture, however: we show that for any group B (of non-ω-measurable cardinality) there is a torsionless group A so that $A^{**}/A \cong B$ (Theorem 4.1). We finish with some remarks about preduals.

Thanks are due to Alan Mekler for his advice and for telling me how to prove Lemma 4.6.

2. PRELIMINARIES

The notation we will use is standard, but we introduce it here to avoid any confusion. If A is a sheaf of Abelian groups over a topological space X, we denote the sections at $U \subseteq X$ by AU. The stalk space we denote by $\Gamma(A)$ with local homeomorphism $p: \Gamma(A) \to X$.

Recall that a topological space is $\mathbf{N}$-compact if every ultrafilter in the Boolean algebra of clopen sets which has the countable intersection property (every countable subset has nonempty intersection) is fixed (the entire ultrafilter has nonempty intersection). The universal zero-dimensional compactification $\beta_0 X$ of a zero-dimensional space X is the space of all clopen ultrafilters with the usual ultrafilter space topology, and the universal $\mathbf{N}$-compactification of X (written $\beta_{\mathbf{N}} X$) is the subspace of $\beta_0 X$ consisting of all those ultrafilters which have the countable intersection property. Note that any discrete space of non-ω-measurable cardinality is $\mathbf{N}$-compact. See [3] for more about $\mathbf{N}$-compact spaces and their use in algebra.

3. Mrówka's Theorem for Sheaves

In [10], Mrówka proved the following theorem.

Definition 3.1. Let X be a topological space and $h: C(X, \mathbf{Z}) \to \mathbf{Z}$ a group homomorphism. We say that a closed subspace $K \subseteq X$ is a *support* for h if $\rho \mid K = 0$ implies $h(\rho) = 0$, for all ρ in $C(X, \mathbf{Z})$.

Theorem 3.2. *A zero-dimensional Hausdorff topological space X is $\mathbf{N}$-compact if and only if every group homomorphism $h: C(X, \mathbf{Z}) \to \mathbf{Z}$ has a compact support.*

Since a set I is $\mathbf{N}$-compact in the discrete topology if and only if its cardinality is not ω-measurable, and a compact subset of a discrete space is finite, the theorem is a generalization of Łoś' result regarding homomorphisms $\mathbf{Z}^I \to \mathbf{Z}$ [6].

We can lift Mrówka's theorem up to the class of groups of global sections of sheaves with simple modifications of the original proof. Recall that the elements of a group of global sections AX may be viewed as the continuous sections of the map $p: \Gamma(A) \to X$. Then in the following, "support" is the same as above.

Theorem 3.3. *A zero-dimensional Hausdorff topological space X is $\mathbf{N}$-compact if and only if for every sheaf A over X and group homomorphism $h: AX \to \mathbf{Z}$, h has compact support.*

Towards proving the theorem we first establish the following lemma, the sheaf-theoretic analogue of a result for groups of continuous functions, cf. [9].

Lemma 3.4. *If B is a sheaf of Abelian groups over a compact zero-dimensional Hausdorff space Y, then any homomorphism $h: BY \to \mathbf{Z}$ has a minimum support K.*

Proof. We first set $S := \{U \subseteq Y \mid U \text{ clopen and a support for } h\}$. Of course, S is nonempty since $Y \in S$. We claim that S is closed under finite intersections. For otherwise, there would be $U_1, U_2 \in S$ and $\rho \in BY$ so that $\rho \mid U_1 \cap U_2 = 0$ but $h(\rho) \neq 0$. Then we could let $\xi \in BY$ be the element such that $\xi \mid U_1 = 0$, $\xi \mid U_2 = \rho \mid U_2$, and $\xi \mid \mathbf{C}(U_1 \cup U_2) = 0$. But then $0 = h(\xi) = h(\rho)$ since U_1 and U_2 are supports. This is a contradiction.

Now set $K := \bigcap S$. Then K is closed, and we show that it is a support. If $\rho \in BY$ has $\rho \mid K = 0$ we know that there is an open set $U \supseteq K$ such that $\rho \mid U = 0$ (a standard fact, see [14]). Since Y is compact and S is closed under finite intersections, there is an element $V \in S$ such that $V \subseteq U$. Then $\rho \mid V = 0$ and hence $h(\rho) = 0$.

Finally, K is contained in every other support for h, since if L is a support, it is an intersection of clopen sets containing it which are then also supports. So $K \subseteq L$. ▮

We can now prove Theorem 3.3.

Proof (Theorem 3.3). We begin with necessity. Suppose that A, h, and X are as given. We may form the zero-dimensional compactification $\beta_0 X$ of X, with inclusion map $j: X \to \beta_0 X$, and then the expansion to $\beta_0 X, j_* A$ (see [14]). This is a sheaf on $\beta_0 X$ with global sections $j_* A(\beta_0 X) \cong AX$. We will not distinguish between the two isomorphic groups, and hence we may consider h to have domain $j_* A(\beta_0 X)$. Now h considered in this way has smallest support K in $\beta_0 X$, by Lemma 3.4; we have only to show that $K \subseteq X$.

Towards a contradiction suppose otherwise, so that there is some $x \in K \setminus X$. Then the set $F = \{U \subseteq X \mid U \text{ clopen}, x \in U^-\}$ is an ultrafilter in X (see [7]). Now F is not fixed, and therefore cannot have the countable intersection property. It follows that there is an increasing sequence of clopen subsets of X, U_n, whose union is X and which are not in F. By virtue of this last, the closure of each of these sets in $\beta_0 X$, U_n^-, is not a support for h. Hence for every $n \in \omega$ there is a section $\rho_n \in AX$ such that $\rho_n \mid U_n = 0$ but $h(\rho_n) \neq 0$.

Now if $(a_n)_{n \in \omega}$ is an element of $\mathbf{Z}^\omega$, we may form the element

$$\sum_{n \in \omega} a_n \rho_n$$

of AX; this is the patch of the elements

$$\sum_{n < k} a_n \rho_n \mid U_k \qquad (k > 0)$$

to X. Then there is a homomorphism $f: Z^\omega \to \mathbf{Z}$ defined by

$$f((a_n)_{n \in \omega}) = h\left(\sum_{n \in \omega} a_n \rho_n\right).$$

The function f has $f(e_n) = h(\rho_n) \neq 0$ for every $n \in \omega$, contradicting the slenderness of $\mathbf{Z}$. (Here e_n is the element which is one in the nth place and zero elsewhere.)

We have established necessity. Sufficiency follows immediately from Mrówka's original result, since the "constant $\mathbf{Z}$" sheaf $\mathbf{Z}_X$ has group of global sections $C(X, \mathbf{Z})$. ▮

Remark. The techniques of the proof differ substantively from Mrówka's original argument only in the use of the sheaf $j_* A$. The object performing the analogous function in Mrówka's proof of Theorem 3.2 is $\mathbf{Z}_{\beta_0 X}$, whereas our proof in this case would use $j_* \mathbf{Z}_X$.

Remark. Although we do not show this here, Theorem 3.3 may be extended to sheaves over frames (locales, complete Heyting algebras). This provides a common generalization of Mrówka's theorem and results of Eda concerning the $\mathbf{Z}$-duals of Boolean products, see [12].

4. The Groups A^{**}/A

In this section we will show how to apply Theorem 3.3 to prove the following result:

THEOREM 4.1. *If A is any group of non-ω-measurable cardinality, there is a torsionless group D so that $D^{**}/D \cong A$.*

Given A, the group D will be a group of global sections of a sheaf. The base space for this sheaf is supplied by the following result, first proved by Eda and Ohta in [2] ([4], [3] also contain proofs.)

LEMMA 4.2. *Suppose that $E \subseteq \omega_1$ is a stationary–costationary subset. Then the following statements hold:*

(i) *$E \cup \{\omega_1\}$ with the topology inherited from the interval topology on $\omega_1 + 1$ is $\mathbf{N}$-compact.*

(ii) *Any compact subset of $E \cup \{\omega_1\}$ is countable.*

(iii) *A (set) function f from E to a discrete space Y is continuous iff $f \mid K$ is continuous for every compact subset $K \subseteq E$.*

In the remainder, E will be a fixed stationary–costationary subset of ω_1 and X will be the topological space $E \cup \{\omega_1\}$. For future reference we note that $\beta_{\mathbf{N}} E = X$. For E is dense in X, X is $\mathbf{N}$-compact, and every point in X is the limit of a unique clopen ultrafilter in E which has the countable intersection property. (The point ω_1 is the limit of the clopen ultrafilter in E which consists of all the unbounded clopen sets).

We will need the following lemma, whose proof we postpone (see below).

LEMMA 4.3. *For any group A, there are groups L and F so that*

$$0 \to L \to F \to A \to 0$$

is a short exact sequence, F is the union of a smooth ω_1-chain of free groups, and $L^* = 0$.

Proof (Theorem 4.1). Suppose A is some Abelian group, of non-ω-measurable cardinality. Take a short exact sequence as supplied by Lemma 4.3, and proceed as follows: First let $\{F_\alpha \mid \alpha \in \omega_1\}$ be an ω_1-filtration of F consisting of free groups. Note that the cardinality of F is not ω-measurable. We give F the discrete topology (so that it is $\mathbb{N}$-compact) and define two sheaves S and T: For $U \subseteq X$ open, let

$$SU = \{f\colon U \to F \text{ continuous} \mid f(\alpha) \in F_\alpha,\ f(\omega_1) \in L(\text{if } \omega_1 \in U)\},$$

$$TU = \{f\colon U \to F \text{ continuous} \mid f(\alpha) \in F_\alpha, \text{ for } \alpha \in \omega_1\}.$$

The restriction maps $SU \to SV$ (for $V \subseteq U$) are just function restriction, and likewise for T. It is easy to check that these are indeed sheaves over X. If K is a compact subset of X, we will denote by SK the sections over K, which we may view as

$$SK = \{f\colon K \to F \text{ continuous} \mid f(\alpha) \in F_\alpha,\ f(\omega_1) \in L(\text{if } \omega_1 \in K)\}.$$

We will need the following observation:

CLAIM 4.4. *The restriction map $SX \to SK$ is onto, for any compact $K \subseteq X$.*

Proof (Claim). If $f \in SK$, we know that there is an open set $U \supseteq K$ so that f extends to U (a standard result; see [8, p. 102]). Since K is compact and X is zero-dimensional, we may take U to be clopen and then extend f further to X by setting it equal to zero on the complement of U. ▮

Now the compact subsets of X form a directed set under containment, and we may define an inverse system of Abelian groups

$$\langle SK, r_{K,M}; K \subseteq M \text{ compact in } X\rangle,$$

where $r_{K,M}$ is the restriction map from SM to SK. The dual of this system, $\langle SK^*, r^*_{K,M}; K \subseteq M \text{ compact}\rangle$ is then a direct system, and we show

$$SX^* \cong \varinjlim SK^*; \qquad K \subseteq X \text{ compact}.$$

The proof of this fact proceeds just as in [2]. The restriction maps $r_K\colon SX \to SK$ are surjective by Claim 4.4. It follows that the maps r_K^* from SK^* to SX^* are injective. The direct limit of these maps $r\colon \varinjlim SK^* \to SX^*$ is then also injective, and we have only to see that it is surjective. Given $h \in SX^*$, let K be its compact support, which exists by Theorem 3.3. Define $h' \in SK^*$ by $h'(\rho) := h(\rho')$, where ρ' is an extension of ρ to X (which exists by Claim 4.4). The fact that K is a support for h guarantees that h' is well defined, and we clearly have $r_K^*(h') = h$. It follows that r is surjective, and altogether that r is an isomorphism.

Now by Lemma 4.2, ω_1 is an isolated point in any compact set $K \subseteq X$ containing it, so we may write

$$SX^* = \varinjlim SK^* \oplus L^* \qquad (K \subseteq E \text{ compact}),$$

$$SX^* = \varinjlim SK^* \qquad (K \subseteq E \text{ compact}).$$

For purely categorical reasons (but see [4, XI.1.3] for a proof),

$$SX^{**} \cong \varprojlim SK^{**} \qquad (K \subseteq E \text{ compact}), \tag{4.2.1}$$

where the inverse limit is taken over the inverse system

$$\langle SK^{**}, r^{**}_{K,M}, K \subseteq M, \text{ compact in } E \rangle.$$

Since any compact subset of E is countable, we may view SK as a subgroup of the group of continuous functions $C(K, F_\alpha)$ for some $\alpha < \omega_1$. As F_α is free, by the Specker–Nöbeling theorem [6, 97.7], we see that SK is free, and since it is of non-ω-measurable cardinality by hypothesis, we conclude that it is reflexive. Then from Eq. (4.2.1) we see that

$$SX^{**} \cong \varprojlim SK \qquad (K \text{ compact in } E),$$

where the inverse limit is taken over the original inverse system

$$\langle SK, r_{K,M}; K \subseteq M \text{ compact in } E \rangle.$$

Now any element of this inverse limit may be viewed as a function from E to F whose restriction to any compact subset K of E is continuous. By Lemma 4.2 we know such functions are continuous. So we may view SX^{**} as the group

$$\{f : E \to F \text{ continuous} \mid f(\alpha) \in F_\alpha\},$$

and, hence, by the comment after Lemma 4.2 we see it as

$$\{f : X \to F \text{ continuous} \mid f(\alpha) \in F_\alpha (\alpha < \omega_1)\},$$

which is precisely TX.

One can easily check that the inclusion of SX into TX commutes with the canonical map from SX to SX^{**}, and that we have $TX/SX \cong A$ via the map $f + SX \to f(\omega_1)$. We conclude that $SX^{**}/SX \cong A$. ∎

As an immediate consequence of Theorem 4.1 we can prove the conjecture of Eklof and Mekler mentioned in the introduction:

COROLLARY 4.5. *Suppose that A is a dual group of non-ω-measurable cardinality. Then there is a group D so that $D^{***}/D^* \cong A$.*

Proof. We may write $A = B^*$, where B of non-ω-measurable cardinality. By Theorem 4.1 there is a group D so that $D^{**}/D^* \cong B$. It follows from Proposition 1.1 that $A = D^{***}/D^*$. ∎

We have only to prove Lemma 4.3, for which we need the following result. A proof of this appears in [1]; we include a different proof here for completeness.

LEMMA 4.6. *There is an $\aleph_1$-free group L of cardinality $\aleph_1$ such that $L^* = \{0\}$, which has a free subgroup of countable rank L_0 so that L/L_0 is $\aleph_1$-free.*

Proof. We will build L as the union of an ω_1-chain $\{L_\alpha \mid \alpha < \omega_1\}$ of $\aleph_1$-free groups. We construct the L_α inductively, ensuring that

(i) L_α is pure in L_β, for $\alpha < \beta$

(ii) L_α/L_0 is $\aleph_1$-free, for any $\alpha < \omega_1$.

(iii) If $a \in L_\alpha$ generates a pure subgroup of L_α then for some $\beta > \alpha$, $\langle a \rangle$ does not split L_β.

To help with the bookkeeping we also inductively define surjective functions $s_\alpha : \omega_1 \twoheadrightarrow L_\alpha$ for each α. Once we have constructed such a chain, its union L will clearly be the desired group.

Our proof will employ an $\aleph_1$-free group B of cardinality $\aleph_1$ which is not a Whitehead group. The existence of such groups was established by Shelah in [13]. (A proof may be also found in [4, p. 229]).

We begin by letting $f : \omega_1 \times \omega_2 \to \mathrm{Succ}(\omega_1)$ be a bijective function such that $f(\delta, \gamma) > \gamma$. Let L_0 be a free group of countable rank, and choose a surjective function $s_0 : \omega_1 \to L_0$. Suppose we have defined L_α and s_α for $\alpha < \beta$. We have two cases:

Case I. β is a limit ordinal. Let $L_\beta = \bigcup_{\alpha < \beta} L_\alpha$. Then L_β is the union of a countable chain of $\aleph_1$-free groups, each pure in the next, and it follows from Pontryagin's criterion that L_β is $\aleph_1$-free. The group L_β/L_0 is $\aleph_1$-free for the same reason. Choose s_β to be any surjective function from ω_1 to L_β.

Case II. $\beta = \alpha + 1$. We know that $f(\delta, \gamma) = \alpha + 1$ for some ordinals δ, γ with $\gamma < \alpha + 1$. If $s_\gamma(\delta)$ does not generate a pure subgroup of L_γ, we set $L_\beta = L_\alpha$. Otherwise, L_β is defined to be the pushout (free amalgamation) as in the following diagram,

$$\begin{tikzcd} 0 \arrow[r] & \mathbf{Z} \arrow[r] \arrow[d, "j"'] & C \arrow[r] \arrow[d, dotted] & B \arrow[r] \arrow[d] & 0 \\ 0 \arrow[r] & L_\alpha \arrow[r, dotted] & L_\beta \arrow[r] & B \arrow[r] & 0 \end{tikzcd}$$

where the top sequence is the witness to the fact that B is not a Whitehead group, and j takes $1 \in \mathbf{Z}$ to $s_\gamma(\delta) \in L_\alpha$.

Now clearly the subgroup generated by $s_\gamma(\delta)$ does not split L_β, and L_α is pure in L_β (since their quotient is torsion-free). To see that L_β is $\aleph_1$-free it suffices to note that $L_\alpha/j(\mathbf{Z})$ is $\aleph_1$-free (which is the case since $\mathbf{Z}$ is embedded as a pure subgroup). Finally, since L_β/L_α is $\aleph_1$-free, and L_α/L_0 is $\aleph_1$-free, we may conclude that L_β/L_0 is $\aleph_1$-free. (An extension of an $\aleph_1$-free group by an $\aleph_1$-free group is $\aleph_1$-free.)

Finally, choose s_β to be a surjection from ω_1 to L_β. This completes the construction. ∎

We may use the group just constructed to establish the following special case of Lemma 4.3.

LEMMA 4.7. *If either B is $\mathbf{Q}$ or Z_{p^∞}, there are groups $L \subseteq F$ so that*

$$0 \to L \to F \to B \to 0$$

is a short exact sequence, F is $\aleph_1$-free of cardinality $\aleph_1$, and $L^* = 0$.

Proof. Given B, let L be a group as supplied in Lemma 4.6, with free subgroup L_0. Let $F_0 \supseteq L_0$ be a free group of countable rank so that $F_0/L_0 \cong B$. We can then form the pushout F, as shown:

$$\begin{tikzcd}
0 \arrow[r] & L_0 \arrow[r] \arrow[d] & F_0 \arrow[r] \arrow[d, dotted] & B \arrow[r] & 0 \\
0 \arrow[r] & L \arrow[r, dotted] & F \arrow[r] & B \arrow[r] & 0
\end{tikzcd}$$

We have only to show that F is $\aleph_1$-free. But if H is a countable subgroup of F, there are countable free subgroups M and G of L and F_0, respectively, so that the following diagram is a pushout:

$$\begin{tikzcd}
L_0 \arrow[r] \arrow[d] & G \arrow[d] \\
M \arrow[r] & H
\end{tikzcd}$$

Then L_0 splits M, since L/L_0 is $\aleph_1$-free, and it follows that H is free. ∎

We may now prove Lemma 4.3, which we restate here.

LEMMA 4.3. *For any group A, there are groups L and F so that*

$$0 \to L \to F \to A \to 0$$

is a short exact sequence, F is the union of a smooth ω_1-chain of free groups, and $L^* = 0$.

Proof. Given A, we may form its divisible hull $D(A)$. We know that $D(A)$ is a sum $\mathbf{Q}^{(I)} \oplus \bigoplus_p Z_{p^\infty}^{(I_p)}$ for some index sets I and I_p, where p ranges over the primes. Each of the summands in $D(A)$ is realizable as in Lemma 4.7. By taking the direct sum of all these sequences, we obtain a short exact sequence

$$0 \to L \to G \to D(A) \to 0$$

with $L^* = 0$. Since G is the direct sum of a set of $\aleph_1$-free groups of cardinality $\aleph_1$, it is the union of a smooth ω_1-chain of free groups. We can take the pullback to obtain a sequence

$$0 \to L \to F \to A \to 0,$$

where F has the same property as G. ∎

Remark. In [4] an example is provided of a dual group which is not a double dual. This example and Corollary 4.5 can be used to construct an illustration of some odd behaviour of the dual functor: Let A be a dual group which is not a double dual. The $A \cong D^{**}/D$ for some dual group D, and in fact $D^{**} \cong D \oplus A$ (Proposition 1.1, and Corollary 4.5). Now D is also a dual group which cannot be a double dual, again by the proposition. So we have an example in which the direct sum of two groups which are duals, but not double duals, is even a triple dual, and an example in which the direct sum of a triple dual need not be even a double dual.

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